Learning About Building Mathematics Instruction from Students' Reasoning: A Professional Development Study

Jana Visnovska
The University of Queensland
<jana.visnovska@gmail.com>

Paul Cobb
Vanderbilt University
<paul.cobb@vanderbilt.edu>

We draw on a 5-year professional development design study to discuss how a group of middle-school mathematics teachers came to view students' reasoning as an instructional resource. The broadening of the professional development goals to include issues of key importance to the teachers played a key role. The study provides guidance in supporting teachers for whom students' reasoning holds little relevance to their work at the outset.

Studies of instructional practices that support all students' mathematical learning with understanding call for teachers to build from their students' reasoning while keeping in mind significant mathematical ideas that are the goal of instruction (Ball, 1993; Gravemeijer, 2004; Lampert, 2001). Instructional approaches of this type require that teachers (a) anticipate and monitor the diverse forms of student reasoning that arise during instruction, (b) decide *which* forms of reasoning need to be further supported with respect to the big ideas of the domain, and (c) envision *how* can students' reasoning be best supported in a classroom (McClain, 2002; Stein, Engle, Smith, & Hughes, 2008). Practices that effectively encompassed these aspects of teacher's work all emphasise students' opportunities to engage in challenging tasks and to communicate their mathematical thinking (Lampert, 2001). Initiatives aimed at supporting investigative teaching approaches in Australia and New Zealand are justified in terms of developing instructional practices of this kind (Cavanagh, 2006; Goos, Dole, & Makar, 2007; Hunter, 2008; Norton, McRobbie, & Cooper, 2002).

The complexity of supporting mathematics teachers to develop instructional practices that build from students' reasoning has been documented by numerous studies of teacher professional development (PD). Researchers have reported that even in cases when teachers seemed engaged in the work-session setting, understanding students' reasoning was not always easy (e.g., Schifter, 2001). In addition, teachers did not always see the relevance of their new insights to their classroom practice (e.g., Cohen, 2004).

Against this background, we report on the final three years of a 5-year PD study[1] in which the participating teachers did not view students' reasoning relevant to their instruction by end of year 3 (Zhao, Visnovska, Cobb, & McClain, 2006). In contrast, when the teachers planned instructional activities at the end of year 5, they routinely envisioned the diverse ways in which students were likely to reason in their classrooms. Our purpose in reporting this case is to outline the actual learning trajectory of the teacher group and to highlight what we learned about supporting middle-school mathematics teachers in coming to see ways in which their students reason as an instructional resource. We document how the initial broadening of the PD goals to include issues that were of key importance from the teachers' perspectives facilitated the learning of the group.

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Background to the PD Design Experiment

The proposed analysis comes from a 5-year PD design study (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) conducted with the group of 12 middle-school teachers. The teachers worked in a diverse urban district in the USA with a high-stakes accountability program. At the beginning of the study, the teachers' instructional practices were typical in the US context in that they attempted to cover the objectives outlined in the State Standards and ensure that their students we on task (Dean, 2005).

We conducted three one-day work-sessions during the first school year, six during each following year, and a 3-day institute during each summer. The PD focused on statistical data analysis and aimed to support the teachers in (a) deepening their understanding of central statistical ideas, (b) making sense of individual students' statistical interpretations and solutions, and (c) adapting instructional sequences[2] developed in prior classroom design experiments (Cobb, 1999; McClain & Cobb, 2001) to their needs and constraints in the classrooms (Cobb & McClain, 2001). The broader research question that we investigated concerned the process of supporting teachers' development of instructional practices in which they place students' reasoning at the centre of their instructional decision-making.

During the first two years of our collaboration, the teachers became increasingly proficient in analyzing statistical data and we began supporting them to adapt the instructional activities in their classrooms. In addition, teaching had become deprivatised and the teachers voiced a desire to engage in joint planning and to observe each other's teaching (Dean, 2005). By the end of the second year, the teachers agreed to videotape their statistics lessons and discuss them in the PD sessions.

Data Sources and Method of Analysis

The data consist of video-recordings of all PD sessions in the years 3-5, field notes of these sessions, copies of the teachers' work, and 7 video-recorded statistics lessons that were viewed by the group (4 in year 3, 3 in year 4). We analyzed patterns and regularities in the teachers' interactions (Dean, 2005) to identify shifts in their pedagogical reasoning as they participated in PD activities across time. In doing so, we used an adaptation of constant comparative method described by Cobb and Whitenack (1996) that involves testing and revising tentative conjectures while working through the data chronologically.

Teachers' Initial Adaptations of Classroom Statistical Activities

During year 3, the major goal of the research team was to support teachers in coming to view students' reasoning as an instructional resource. We considered this goal feasible given that (a) the nature of teachers' participation in the sessions allowed for genuine discussions of the pedagogical problems that they encountered in their classrooms (Dean, 2005), and (b) the teachers collaboratively planned and analyzed lessons that they enacted in their classrooms on regular basis. In pursuing the goal, we asked the teachers to categorise solutions evident in their students' written work and to propose how could they use these when planning for subsequent instruction. In doing so, we attempted to orient the teachers *directly* to focus on their students' reasoning and how could they build on it in their classrooms.

We remained largely unsuccessful in these efforts throughout year 3. Although the teachers became increasingly proficient in categorising their students' solutions, they

[2] Descriptions of and justifications for the instructional sequences can be found in Cobb & McClain (2004).

struggled with the idea of using this knowledge to plan subsequent instruction. Wayne's spontaneous reflection captured the group's struggles during session five of this year:

Wesley:	I am very uncomfortable.
Researcher:	With what?
Wesley:	… I am very uncomfortable with my lack of understanding of my answer to the question "What is there that I can build on?" … I am just trying to make a statement that I am not sure I've got it. … But I am just not sure, I feel.
Researcher:	And I think you're not alone. For what I can make out. Is that fair? [Muriel, Erin, and Kate who faced the researcher nod.]
Lisa:	The kids were good and they've done it [the task]. That's good isn't it?
Wesley:	I mean I feel good about what my kids did and I would be confidently walking to the classroom the next day and not feel like I was creating educational malpractice.
Researcher:	No, let me put this in perspective. … We're talking about something that is *way* out there.
Muriel:	And we aren't there yet either. [Teachers laugh.]

Zhao's (in preparation) retrospective analysis of teachers' use of student work in these sessions indicates that the teachers based their categorisations on retrospective *assessments* of students' proficiency in using various methods of statistical analysis (e.g., using totals to compare two datasets). Collecting student work indicated to the teachers that the learning phase of a lesson has already concluded and that they now needed to assess individual students' accomplishments. This teachers' interpretation was reasonable within the requirements of their schools. Our implicit assumption that the group would constitute the activity as prospective planning of instruction did not prove viable.

Throughout year three, the normative view within the teacher group was that student learning occurred primarily when they completed instructional activities in small groups, but not when they participated in the subsequent whole class discussion of their solutions. During small group work, the teachers intervened routinely as students conducted their analyses. In contrast, their interventions during the concluding class discussions were minimal. The normative justifications for devoting instructional time to student presentations included that (a) students learn easier from their peers than from the teacher, and (b) presentations help students improve their social skills, support the development of their self-esteem, and provide a rationale to produce solutions.

At the same time, most of the teachers struggled to help their students listen to their peers' presentations. We conjectured that by focusing on the reasoning of the students in the video-recorded lessons, we would help the teachers realise that many of the listening students could not understand their classmates' explanations. We therefore pressed the teachers to consider "What sense did listening students make from presentations?", "What did listening students *hear*?", and "What was there that was worth to listen to?" Throughout all 4 video analyses in year three, the teachers' interpretations of our orienting questions differed significantly from our intent in that they did not revolve around the sense that students were making of others' explanations, but instead centred on *instructional strategies* that were or could have been used by the teacher in the lessons.

Our repeated failures in enabling the teachers to examine classroom instruction from a student's point of view indicate that adopting such perspective involved a profound shift for the teachers. Our attempts at reorienting video analyses resulted in teacher dissatisfaction with the choice of focal issue and in complaints that we have already "done the same thing" many times. It became obvious that we could not continue to directly press for the focus on students' reasoning, as the teachers did not find this focus instructionally relevant. At the same time, we became convinced that the ability to *decentre* and examine instruction from a student's perspective is key in proactively supporting students' learning.

Broadening the Focus: Issues of Students' Motivation and Engagement

In designing the 3-day summer workshop conducted at the end of year 3, we planned to continue supporting the teachers in shifting their focus from the teacher's actions to the sense that students might have been making of classroom activities. However, rather than doing so in context of students' reasoning, we sought issues that the teachers already considered instructionally important. Students' motivation provided such a focus.

An analysis of classroom lessons and follow-up interviews revealed that all the teachers viewed students' motivation to be a major determinant of both students' engagement in class and mathematical learning (Zhao et al., 2006) However, ways in which teaching resulted in students' learning were largely a black box for the teachers. Whether students learned or not depended to a great extent on their motivation, which the teachers attributed to societal and economical factors out of their control. Student motivation and engagement thus played out as highly problematic issues in the teachers' daily instruction.

We conjectured that it would be beneficial to support teachers in (a) coming to view students' motivation as situational and within their control by re-framing this issue as one of students' mathematical interests, and in (b) coming to view the cultivation of students' mathematical interests as an important goal of instruction (cf. Dewey, 1913/1975). To accomplish this, we planned to engage the teachers in investigating how students' classroom experiences contributed to development of their mathematical interests during the prior statistics classroom design experiment. We conjectured that in attempting to understand what became interesting for the students in the statistics class, the teachers would adopt a student's perspective. We further conjectured that this would later allow us to focus on students' reasoning as an aspect of instruction that is relevant in cultivating students' mathematical interests. The conjectured means of supporting the group's learning included a series of activities in which the teachers analyzed (a) interview excerpts in which students described their learning experiences and valuations of those experiences in the statistics class and in an algebra class in which they were enrolled at the same time, and (b) a series of videos from the classroom design experiment.

At various points of the 3-day workshop, all teachers indicated that they found issues of students' motivation very relevant to their instruction, appreciated this focus, and felt they were learning a lot. After analyzing students' interview excerpts, the teachers questioned their prior view that some students were inherently unmotivated. The group concluded that the same students who were unmotivated in the algebra class appeared to become highly motivated and to view themselves as competent in the statistics class. The teachers therefore found it worthwhile to analyze interactions in the statistics class in order to understand how the students became interested in analyzing data. They spent almost 4 hours analyzing classroom video in small groups, including one lunch break. They were fully aware of the change in their views about students' motivation, stating that they should be able to shape students' engagement, and needed to learn how to do it. Based on video analyses, they elaborated views of students' participation that highlighted students' ability to make insightful contributions rather than the appearance of paying attention.

Crucially, the teachers considered the videos that they analyzed to be examples of *good instruction*. They made this interpretation because the instruction (a) was aligned with the discourse of reform current in their school, (b) was enacted in a real classroom with a diverse group of students, and (c) surpassed both their own and their colleagues' efforts to support student learning. The teachers did not raise critique during the activities.

Despite the significant progress that the group made with respect to the issue of student motivation, we found that supporting the teachers to decentre in this context was not trivial. When the teachers discussed what they learned about cultivating students' interests in analyzing data, they (a) proposed labels that they viewed as representing "reform math" (e.g., "safe learning environment"), (b) used examples from video to justify the pre-chosen categories, and in doing so, (c) adopted an observer's (rather than a student's) viewpoint. However, when we pressed the teachers to elaborate on their rather general contributions, at least 5 out of the 9 participating teachers began to adopt a student's perspective:

Researcher: [Following up on "Make me and others clarify ideas" bullet reported by a teacher group] How does it help me to learn? [When] somebody makes me to clarify my ideas.

Brian: If she [the teacher] just lets me slide by with a poorly expressed idea then, for one, nobody is really sure if I understood it in the first place. But more importantly, all the other students will never get to fully appreciate what my idea was. … [The teacher] tries to make sure that the rest of the class has at least had a chance to appreciate that argument.

Data Generation Discussions and Students' Interests

In year 4, we extended our efforts to support the teachers in making students' reasoning a relevant aspect of teaching. We continued to build on the themes of motivation and cultivating students' interests. All the teachers initially proposed that the initial *data generation* discussion in which they introduced the instructional task was responsible for students' interest. They assumed that in order to be of interest, students needed to have prior personal experience with the topic at hand. As a result, the teachers did not view topics of broader social significance (e.g., treating AIDS) as potentially interesting to *their* students. These teachers' views contrasted with the data from the classroom experiments where scenarios of this type proved to be the most productive (Hodge, Visnovska, Zhao, & Cobb, 2007).

We intended to support the teachers in realising that (a) purposes of data generation discussion extended beyond enticing students' momentary interest in a single task, and that (b) other aspects of the lesson also contributed to students' development of interests in analysing data. To support these shifts, we engaged the teachers in an intentionally flawed data generation discussion. For instance, we first orchestrated a lively discussion of sleeping problems with which teachers had personal experience, and then asked them to analyze data on the effectiveness of two kinds of sleep medications without clarifying what was measured and how. The teachers' difficulties in interpreting the data and their frustrations opened opportunities for the group to reflect on the role of data generation discussion in both making analysis possible and in supporting students' interest and engagement while they analyzed data.

It subsequently became routine to frame discussions of teachers' classroom videos by asking *what would their students need to understand* about the data in order to analyze it productively. Throughout these discussions, the teachers adopted a student's perspective and noted that that their students became interested in activities even when the problem scenario was not based on their personal experience. Four out of the 12 participating teachers commented that they now actively worked to help their students develop ways of relating to the problem scenario during data generation discussions, rather than expecting students to relate to it at the outset. The normative purposes for data generation discussion in the teacher group started to include clarifications of both (a) significance of the problem at hand—as a means to *engage* students—and (b) how an experiment could be designed and what aspects of situation should be measured—as a means of making analysis and *students' interests in it* possible.

By the end of year 4, some of the teachers started to view the concluding class discussions in which students justified their analyses as a source of students' developing interests. We further supported these views by engaging the teachers in statistical analyses that they found conceptually demanding (e.g., bivariate data analyses) and by subsequently reflecting on why they were willing to discuss solutions even after they had all agreed on the answer to the question at hand. The group concluded that they found the discussions worthwhile because in understanding others' analyses, they gained additional insights in the situation. In this way, the distinction between students' initial pragmatic interests in addressing the question at hand and the development of statistical interests became visible in the group.

Data Analysis Discussions and Students' Reasoning

The teachers first began to adopt a students' perspective in the context of the concluding data analysis discussions to determine if students were interested in particular instructional activities. Although it continued to be difficult for the teachers to focus on the listening students, the teachers' interpretations of guiding questions now aligned with our intent. At least 4 teachers suggested that if the listening students did not understand their classmates' explanations, they could not gain new insights and would not remain engaged. The teachers now wanted to investigate why some students in video could not understand others' explanations. In addition, several teachers started to proactively support the listening students in their classrooms. In session 4, Ben shared:

Ben: When we discuss, kids would say it [solution] in their own words, and I knew what they meant, but I could tell a lot of the other kids [did not know]: "What are you talking about?"

Researcher: What did you do in that situation?

Ben: I tried to rephrase what they were saying a little bit, I asked them if they could—at first I'd say "What do you mean?" and try and get them to say it. But when they couldn't explain it well, I would then try "Is this what you are saying?" Usually I know what they are saying, but most of the other kids don't. And they [presenters] know what they mean; they just have trouble verbalizing it. My kids wanted to have a debate about this. And I've never done a debate. I thought it was really interesting… But they were having trouble doing that.

Understanding the details of students' reasoning became gradually constituted as a means of learning about instruction and collectively working on its improvement in the group.

In the summer session at the end of year 4, we engaged the teachers in a series of activities designed to support them in reconstructing the rationale for the statistics instructional sequence in terms of students' reasoning. In these activities, the teachers analyzed a series of videos from the design experiment classroom in order to document the diversity in students' reasoning at each phase of the sequence. We conjectured that the group would produce a record of the shifts in students' reasoning and, in doing so, would adopt a student's perspective.

The teachers were highly engaged when they shared findings. They did not attempt to fit in pre-determined reform-like categories but instead debated the details of students' reasoning. They were often critical of the video teacher by citing episodes in which she missed opportunities to support students' discussions. In addition to describing students' solutions, the teachers also identified the big statistical ideas that underpinned these solutions. Nevertheless, we needed to provide substantive support in teasing out from the teachers' analyses (a) broad commonalities in the diverse solutions produced in each phase of the sequence, and (b) the shifts in ways that students reasoned in different phases of the sequence. The teachers found the organisation we proposed (Figure 1) reasonable and used it effectively to structure the reports of their analyses. They made sense of the proposed shifts in terms of "progression of difficulty", "conceptual levels", and "Van Hiele

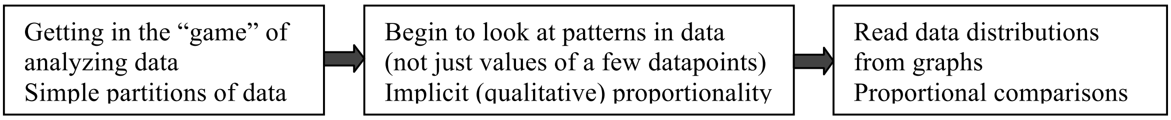

Figure 1. Initial shifts in students' reasoning.

categories but not so formalized". However, our press for keeping in mind what the above labels meant in terms of specific forms of students' reasoning was only partially successful. We conjectured that in order to use this framing productively, the teachers needed opportunities to explore what guidance could it provide in their instruction. We designed opportunities of this type during the final, fifth year of the PD experiment.

Students' Reasoning and Instructional Planning

We conceived of year 5 as a performance assessment the goal of which was to understand what the teachers had learned and the role that the PD had played in supporting that learning. The teachers reviewed and critiqued two sets of instructional units in statistics, and selected and adapted tasks that they found most suitable for their classrooms. The teachers' participation in these activities indicated that anticipating the ways in which students might reason during specific instructional activities was now relevant to planning, at least when the focus was on statistical data analysis (Visnovska, Zhao, & Gresalfi, 2007). The teachers justified the choice of tasks by specifying the forms of reasoning that they hoped would emerge in the classroom. In doing so, they drew substantially on the *shifts in students' reasoning* framework collectively developed during year 4 summer session.

Discussion

Our goal in outlining the actual learning trajectory of a group of mathematics teachers was to illustrate that in order to support their learning effectively, we had to both (a) build from the teachers' current concerns and practices and (b) support the subsequent sifts in the ways that the group reasoned about instruction towards the big goal of using students' reasoning as an instructional resource (cf. Ball, 1993). Among the important lessons we learned was that a direct focus on students' reasoning from the outset in PD might often be ineffective, especially when students' reasoning is not a part of the vision for high quality mathematics instruction promoted in the teachers' schools. In case of our study, students' reasoning initially appeared irrelevant to the issues for which school leaders held the teachers accountable.

We also learned to capitalise on the difficulties that teachers encounter as they attempt to entice students they perceive as unmotivated to engage in instruction. The outlined trajectory indicates how we used these concerns to support the learning of a teacher group without abandoning the PD agenda. Indeed, the initial focus on issues of students' motivation enabled us to support the teachers in reshaping their views of a high quality mathematics instruction as well as of the means by which it can be developed. The trajectory is justified in terms of teachers' actual learning and the means by which it was supported. This justification enables it to be adapted to new settings, and thus provides resources for the generalisation of design research findings by means of explanatory framework (Steffe & Thompson, 2000).

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